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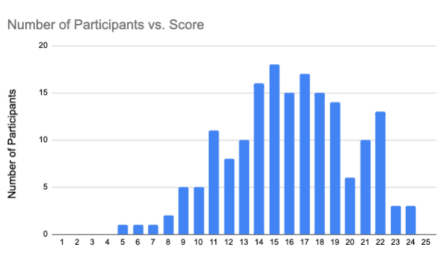
Carlson et al. (2010) developed the 25-item multiple choice Precalculus Concept Assessment (PCA) to investigate reasoning abilities and meanings researchers (e.g., Carlson et al., 2002; Dubinsky & Harel, 1992; Oehrtman et al., 2008; Thompson & Silverman, 2007) have established as critical for pre-calculus and calculus learning. Since its initial development and validation, the PCA has been administered to thousands of secondary and post-secondary students, providing key insights into their reasoning abilities, as well as their potential success (as measured by grades) in future calculus courses. For instance, using a population of 248 students who were entering a first-semester calculus course taught by six different instructors, Carlson et al. (2010) identified that 77% of the students who scored 13 or higher passed the course (i.e., C or better). Meanwhile, 60% of the students who scored 12 or lower failed (i.e., D, F, or W). The authors illustrated that the correlation between course grades and PCA score were as strong or stronger than other popular educational math tests including the MAA placement test.

Given the insights the PCA provides relative to pre-calculus and calculus students, we grew interested in the extent the PCA can provide useful insights with other populations. Across several semesters, we administered the PCA to 174 undergraduate students upon their entry to a secondary mathematics teacher preparation program. Their program entry typically occurs during their sophomore or junior year of undergraduate studies, and after having taken two math courses beyond a calculus sequence. In this poster, we present on the results of that administration. Specifically, we report on an analysis of the aggregate scores of the population (Figure 1a), as well as their performance on covariational reasoning items (Figure 1b). With respect to the covariational reasoning item performance, we draw on our expertise and research (Moore, 2021; Moore et al., 2022; Moore et al., 2019) to develop hypotheses regarding discrepancies in performance. Specifically, we highlight differences between items based on the level of covariational reasoning targeted by the item. We also explore differences between items based on the items' figurative material and the extent the material afforded enacting quantitative operations versus numerical operations. These differences provide potential insights and implications relative to the participants' grounding to reason about and teach for key concepts of secondary mathematics.



(a)

Covariational Reasoning Items

Question	Correct	Incorrect	% Correct	% Wrong
7	101	73	58.05%	41.95%
8	50	124	28.74%	71.26%
15	70	104	40.23%	59.77%
18	140	33	80.46%	19.54%
19	125	49	71.84%	28.16%
22	74	100	42.53%	57.47%
24	134	40	77.01%	22.99%
25	81	92	46.55%	53.45%

(b)

Figure 1. (a) PCA Participant Scores and (b) Covariational Reasoning Item Performance.

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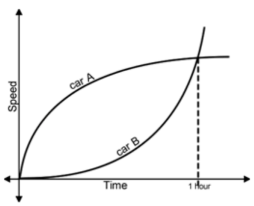
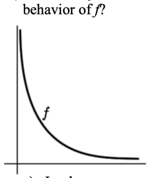
The given graph represents speed vs. time for two cars. (Assume the cars start from the same position and are traveling in the same direction.) Use this information and the graph below to answer item 8.



8) What is the relationship between the ***position*** of car A and car B at $t = 1$ hr.?

a) Car A and car B are colliding.
b) Car A is ahead of car B.
c) Car B is ahead of car A.
d) Car B is passing car A.
e) The cars are at the same position.

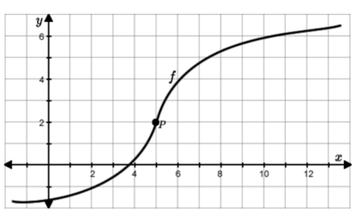
24) A function *f* is defined by the following graph. Which of the following describes the behavior of *f*?



I. As the value of x approaches 0, the value of f increases.
II. As the value of x increases, the value of f approaches 0.
III. As the value of x approaches 0, the value of f approaches 0.

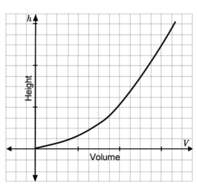
a) I only
b) II only
c) III only
d) I and II
e) II and III

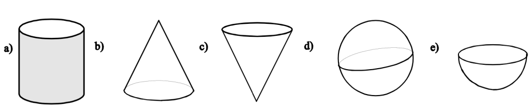
19) Using the graph below, explain the behavior of function *f* on the interval from $x = 5$ to $x = 12$.



a) Increasing at an increasing rate.
b) Increasing at a decreasing rate.
c) Increasing at a constant rate.
d) Decreasing at a decreasing rate.
e) Decreasing at an increasing rate.

15) The following graph represents the height of water as a function of volume as water is poured into a container. Which container is represented by this graph?

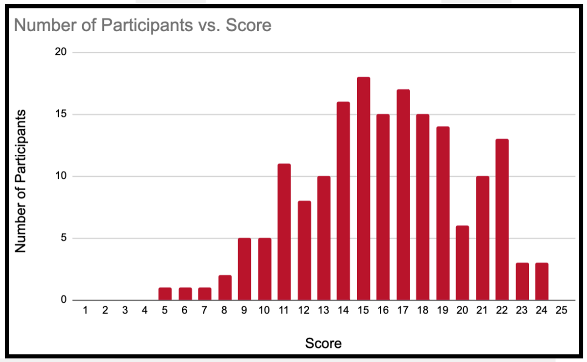




Differences in responses to covariation items provide insights into their covariation meanings.

The prospective teachers have formed indexical associations between graphs and covariation statements, but that does not imply their having abstracted graphs in terms of re-presenting quantitative and covariational operations.

MATCHING GAME



PAPER

Match The Task to the Answer Data Set

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SET ONE

Ans.	#	%
a	4	2.3%
b	20	11.49%
c	1	0.57%
d	134	77.01%
e	15	8.62%

SET TWO

Ans.	#	%
a	11	6.32%
b	50	28.74%
c	3	1.72%
d	7	4.02%
e	103	59.20%

SET THREE

Ans.	#	%
a	5	2.87%
b	70	40.23%
c	72	41.38%
d	3	1.72%
e	24	13.79%

SET FOUR

Ans.	#	%
a	44	25.29%
b	125	71.84%
c	5	2.87%
d	0	0%
e	0	0%